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Let $\frac{\rho}{h}\sin A\cos(\delta-A)+p/c=D$, $\frac{\rho}{h}\sin(\delta-A)=E$.

$\therefore rD-p=\pm E\sqrt{(h^2-r^2\sin^2A)}$.

$(D^2+E^2\sin^2A)r^2-2Drp=E^2h^2-p^2$.

$\therefore r=\frac{Dp}{D^2+E^2\sin^2A}$, by differentiation.

$\therefore D^2h^2+E^2h^2\sin^2A=p^2\sin^2A$.

Substituting D and E and reducing $\rho^2c^2\sin^2A+2\rho chp\sin A\cos(\delta-A)$ $=p^2c^2\sin^2A-p^2h^2$.

$\therefore \rho^2+2\rho H\cos(\delta-A)=G$ is the equation sought, and represents a circle.

Also solved by F. P. Matz.

243. Proposed by F. P. MATZ, Sc. D., Ph. D., Reading, Pa.

What is the equation to the curve on which lie the centers of the inscribed circles in the right-angled triangles of hypotenuse h?

I. Solution by G. W. GREENWOOD, M. A. (Oxon), Lebanon, Ill.

Take as axes the hypotenuse and its perpendicular bisector. Denoting the length of the hypotenuse by h, and one acute angle by α, the equations to the bisectors of the acute angles may be written

$$y=(x+\tfrac{1}{2}h)\tan\tfrac{1}{2}\alpha,$$
$$y=-(x-\tfrac{1}{2}h)\tan(\tfrac{1}{4}\pi-\tfrac{1}{2}\alpha),$$
$$=-(x-\tfrac{1}{2}h)\frac{1-\tan\frac{1}{2}\alpha}{1+\tan\frac{1}{2}\alpha}.$$

By eliminating α between these equations we get as the locus of their intersection, that is, of the center of the inscribed circle, $4(x^2+hy+y^2)=h^2$.

II. Solution by L. S. SHIVELY, Mt. Morris College, Mt. Morris, Ill.

It is evident that the angle subtended by the hypotenuse and with vertex at the center of the inscribed circle equals 135°. Since it is constant and equal to 135°, its vertex, and hence the center of the circle, lies upon that arc of a circle constructed upon h as a chord, to contain an angle of 135°. Construct such a circle and join O, its center, with A and B, the ends of h. Then $\angle AOB=90°$. Also $OA=\frac{1}{2}h\sqrt{2}$. Referred to O as origin of coördinates, the equation of the circle is $2x^2+2y^2=h^2$.

Also solved by J. Scheffer, E. L. Sherwood, G. B. M. Zerr, and Elmer Schuyler.

244. Proposed by O. W. ANTHONY, Head of the Mathematical Department, DeWitt Clinton High School, New York.

Upon the sides of a triangle as bases isosceles triangles with base angles of 30° are constructed. Show that the lines joining the vertices of these isosceles triangles form an equilateral triangle.

I. Solution by **H. M. ARMSTRONG**, Cooch's Bridge, Delaware.

Let the sides of the given triangle be denoted by the vectors 2α, 2β, and $2(\alpha+\beta)$. Let i denote a unit vector perpendicular to the plane of the triangle. Then

$$\rho_1=\alpha+\frac{1}{\sqrt{3}}i\alpha \text{.........} (1),\quad \rho_2=2\alpha+\beta+\frac{1}{\sqrt{3}}ei\beta \text{.........} (2),\quad \rho_3=\alpha+\beta-ei(\alpha+\beta) \text{.........} (3)$$

are the vectors drawn to the vertices of the three isosceles triangles.

Hence the vectors of the sides of the new triangle are obviously $\rho_2-\rho_1$, $\rho_3-\rho_2$, $\rho_1-\rho_2$. By means of (1), (2), and (3), we find that $(\rho_2-\rho_1)^2=(\rho_3-\rho_2)^2=(\rho_1-\rho_2)^2$.

Hence $T(\rho_2-\rho_1)=T(\rho_3-\rho_2)=T(\rho_1-\rho_2)$. Therefore the triangle is equilateral.

II. Solution by **G. W. GREENWOOD, M. A.** (Oxon). Lebanon, Ill.

Denote the vertices by A, B, C, and the remaining vertices of the triangles adjacent to BC, CA, AB, by A', B', C', respectively. I will assume that the triangles are all exterior to the given triangle. Then

$$A'C=a/\sqrt{3},\quad B'C=b/\sqrt{3},\quad \angle A'CB'=60^\circ+C.$$
$$\begin{aligned}A'B'^2&=\tfrac{1}{3}[a^2+b^2-2ab\cos(60+C)]\\&=\tfrac{1}{3}[a^2+b^2-ab(\cos C-\sqrt{3}\sin C)]\\&=\tfrac{1}{6}[a^2+b^2+c^2+2\sqrt{3}\,ab\sin C]\\&=\tfrac{1}{6}[a^2+b^2+c^2+\sqrt{3}\,abc/r],\end{aligned}$$

where r is the radius of the circumcircle of the triangle ABC. The result shows that $A'B'=B'C'=C'A'$. The same result will follow if A' and A are on the same side of BC, etc.

Also solved by M. E. Graber, J. Scheffer, and G. B. M. Zerr. G. I. Hopkins refers to solution of Problem 96, Vol. 5, No. 4.

245. Proposed by **J. H. M. MACLAGAN-WEDDERBURN**, M. A., Chicago, Ill.

Given two lines of fixed length, $AB=a$, $BC=b$, perpendicular to each other. A line CP is drawn making $\angle BPC=\theta$. AD is the perpendicular to AB meeting CP in D. Find by Euclidean construction the angle θ such that $AD^2\cos^2\theta+b^2\sin^2\theta$ is a minimum.

Solution by the **PROPOSER**.

Draw any line CP. From B drop the perpendicular BL upon CP; draw AM parallel to PC meeting BL in M. Join CM. Then $CM^2=CL^2+LM^2=b^2\sin^2\theta+AD^2\cos^2\theta$. The locus of M is a circle described on AB as a diameter with O, the middle point of AB, as center. In order that CM^2, and therefore CM shall be a minimum, the line CM must pass through O.

The construction is therefore the following: Bisect AB in O; produce BOA to P so that $OP=OC$. Then BPC=required angle θ. [Newton's *Principia*, Vol. 2, Section 7, Proposition XXXIV, Scholium.]